

Size quantization of Dirac fermions in graphene constrictions

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Quantum point contacts (QPCs) are cornerstones of mesoscopic physics and central building blocks for quantum electronics. Although the Fermi wavelength in high-quality bulk graphene can be tuned up to hundreds of nanometers, the observation of quantum confinement of Dirac electrons in nanostructured graphene systems has proven surprisingly challenging. Here we show ballistic transport and quantized conductance of size-confined Dirac fermions in lithographically-defined graphene constrictions. At high charge carrier densities, the observed conductance agrees excellently with the Landauer theory of ballistic transport without any adjustable parameter. Experimental data and simulations for the evolution of the conductance with magnetic field unambiguously confirm the identification of size quantization in the constriction. Close to the charge neutrality point, bias voltage spectroscopy reveals a renormalized Fermi velocity ($v_F \approx 1.5 \times 10^6$ m/s) in our graphene constrictions. Moreover, at low carrier density transport measurements allow probing the density of localized states at edges, thus offering a unique handle on edge physics in graphene devices.

The observation of unique transport phenomena in graphene, such as Klein tunneling¹, evanescent wave transport², or the half-integer^{3,4} and fractional^{5,6} quantum Hall effect are directly related to the material quality as well as the relativistic dispersion of the charge carriers. As the quality of bulk graphene has been impressively improved in the last years^{7,8}, the understanding of the role and limitations of edges on transport properties of graphene is becoming increasingly important. This is particularly true for nanoscale graphene systems where edges can dominate device properties. Indeed, the rough edges of graphene nanodevices are most probably responsible for the difficulties in observing clear confinement induced quantization effects, such as quantized conductance⁹ and shell filling¹⁰. So far signatures of quantized conductance have only been observed in suspended graphene, however with limited control and information on geometry and constriction width¹¹. More generally, with further progress in fabrication technology, graphene nanoribbons and constrictions are expected to

evolve from a disorder dominated^{12–15} transport behavior to a quasi-ballistic regime where boundary effects, crystal alignment, and edge defects^{16,17} govern the transport characteristics. This will open the door to investigate interesting phenomena arising from edge states, including magnetic order at zig-zag edges¹⁸, an unusual Josephson effect, unconventional edge states²⁰, magnetic edge-state excitons²¹ or topologically protected quantum spin Hall states²².

In this work we report on the observation of quantum confinement and edge states in ballistic transport through graphene constrictions approximating quantum point contacts. We prepared 4-probe devices based on high-mobility graphene-hexagonal boron nitride (hBN) sandwiches on SiO₂/Si substrates and use reactive ion etching to pattern narrow constrictions (see Methods) with widths ranging from $W \approx 230$ to 850 nm, connecting wide leads (Figs. 1a-1c). The graphene leads are side-contacted⁸ by chrome/gold electrodes. A back gate voltage is applied on the highly doped Si substrate to tune the carrier density in the graphene layer, $n = \alpha(V_g - V_g^0) = \alpha\Delta V_g$, where α is the so-called lever arm and V_g^0 is the gate voltage of the minimum conductance, i.e. the charge neutrality point. To demonstrate the high electronic quality of our graphene-hBN sandwich structures we show the gate characteristic of a reference Hall bar device (Fig. 1d). From this data we extract a carrier mobility in the range of around 150.000 cm²/Vs (see Supplementary Note 1), resulting in a mean free path exceeding 1 μ m at around $\Delta V_g = 4.6$ V. Thus, the mean free path is expected to clearly exceed all relevant length scales in our constriction devices giving rise to ballistic transport.

RESULTS

Ballistic transport.

We measure the conductance as function of gate voltage for a number of constrictions with different widths W (Fig. 1d; see labels in Fig. 1e). The observed square root dependence $G \propto \sqrt{\Delta V_g} \propto \sqrt{n}$ (see dashed lines in Fig. 1d) is a first indication of highly ballistic transport in our devices. Indeed, according to the Landauer theory

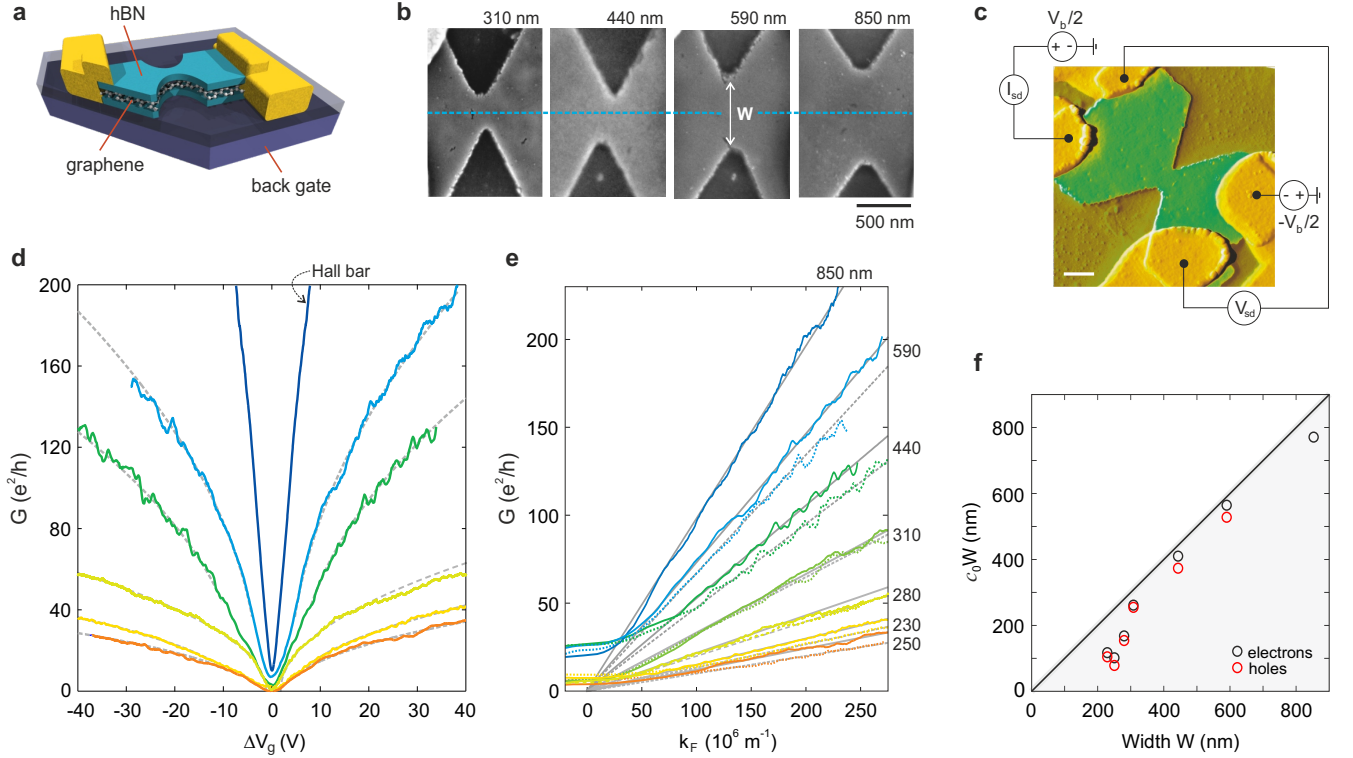


Figure 1. Width dependent ballistic transport in etched graphene nanoconstrictions encapsulated in hBN. **a**, Schematic illustration of a hBN-graphene sandwich device with the bottom- and top-layers of hBN appearing in green, the gold contacts in yellow, the SiO₂ in dark blue and the Si back gate in purple. **b**, Scanning electron microscope (SEM) images of four investigated graphene constrictions patterned using reactive ion etching. **c**, False colored atomic force microscope (AFM) image of a fabricated device. Transport is measured in a four-probe configuration to eliminate any unwanted resistance of the one-dimensional contacts⁸. The yellow color denotes the gold contacts, green the top layer of hBN and brown the SiO₂ substrate. The white scale bar represents 500 nm. **d**, Low-bias back-gate characteristics of a Hall bar device (see arrow) and of five constriction devices with different widths ranging from 850 to 230 nm (color code as in panel e). The dashed grey lines are fits to the data. **e**, Low-bias four-terminal conductance of graphene quantum point contacts as function of k_F extracted in the high carrier density limit for seven different samples. The color encodes the different samples with different constriction widths (see labels). Grey lines represent a linear fit at high values of k_F , inserted as guide to the eye. Conductance deviates from the expected linear slope for small k_F . Electron (hole) transport is plotted as solid (dashed) line. Data are taken at temperatures below 2 K. **f**, Comparison of $c_0 W$ from conductance traces (panel e) with the width W (extracted from SEM images).

for ballistic transport, the conductance through a perfect constriction increases by an additional conductance quantum e^2/h whenever Wk_F reaches a multiple of π ,

$$G = \frac{4e^2}{h} \sum_{m=1}^{\infty} \theta\left(\frac{Wk_F}{\pi} - m\right), \quad (1)$$

where $k_F = \sqrt{\pi n}$ is the Fermi wave number, the factor four accounts for the valley and spin degeneracies, θ is the step function, and we have neglected minor phase contributions due to details of the graphene edge²³ for simplicity. Fourier expansion of Eq. (1) yields

$$G = \frac{4e^2}{h} \frac{c_0 W k_F}{\pi} + \frac{4e^2}{h} \left[\sum_{j=1}^{\infty} c_j \sin(2jWk_F - \phi_j) - \frac{c_0}{2} \right]. \quad (2)$$

For an ideal constriction $c_0 = 1$, $\phi_j = 0$, and $c_j = 1/(j\pi)$, $j > 0$. In the presence of edge roughness, c_0 is reduced

to a value below 1 due to limited average transmission, and higher Fourier components c_j are expected to decay in magnitude and acquire random scattering phases $\phi_j \neq 0$. Consequently, the sharp quantization steps turn into periodic modulations as will be shown below. Averaged over these modulations only the zeroth order term in the expansion [Eq. (2)] survives. This mean conductance $G^{(0)}$ of a constriction of width W thus features a linear dependence on k_F , or, equivalently, a square-root dependence as a function of back-gate voltage assuming an energy-independent transmission c_0 of all modes, in accord with Fig. 1d. By measuring the carrier density dependent quantum Hall effect at high magnetic fields^{24,25}, we can independently determine the gate coupling α for each device (see Supplementary Note 2). We can thus unfold the dependence on V_g and study both the electron and hole conductance as function of k_F (Fig. 1e). From the linear slopes of $G(k_F)$, the product $c_0 W$ can be

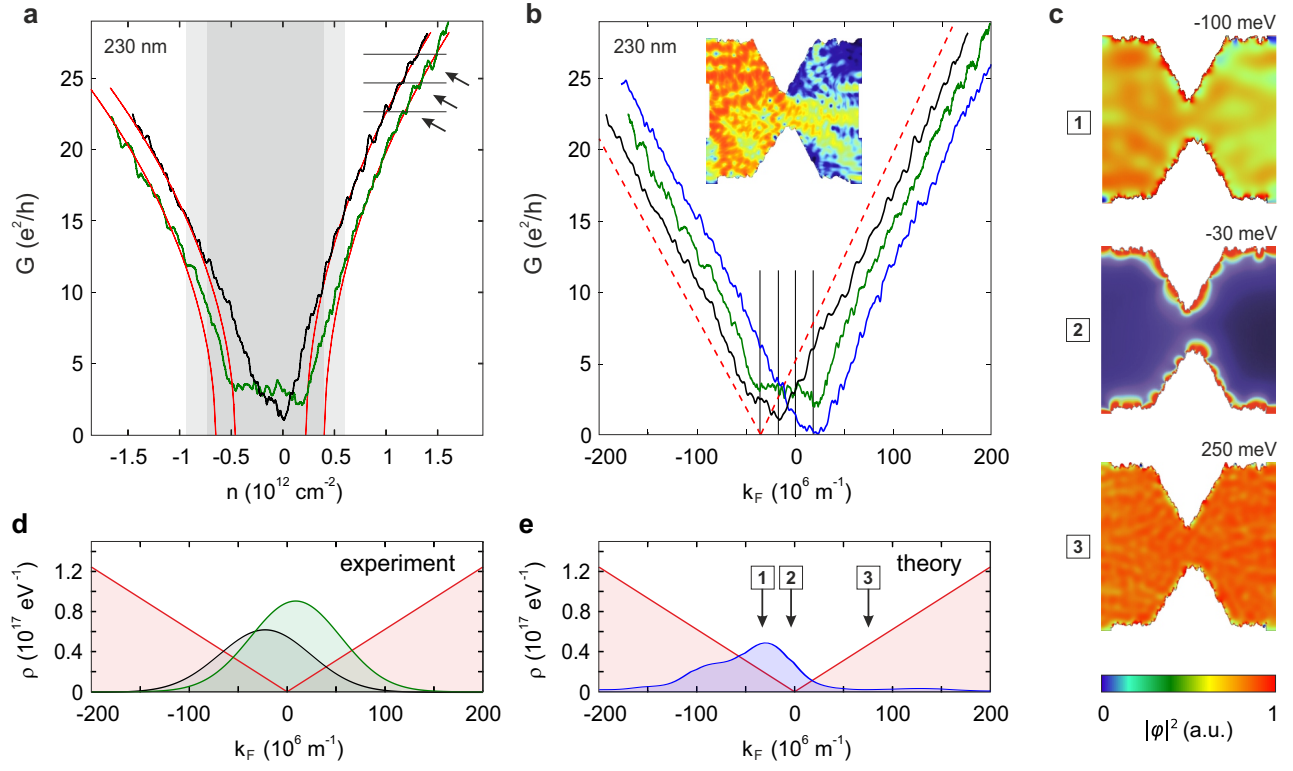


Figure 2. Conductance through graphene quantum point contacts **a**, Conductance traces of two different cool-downs (black and green curve) of the same constriction ($W \approx 230$ nm) as a function of charge carrier density. For the black (green) cool-down, shaded gray (light gray) regions denote deviations from the ideal Landauer model $G \propto \sqrt{n}$ shown in red. At higher conductance values we observe well reproduced ‘kinks’ with spacings on the order of $2e^2/h$ (see arrows and horizontal lines). **b**, Experimental conductance trace as a function of k_F after correction for the density of trap states (black and green curves) and theoretical simulations of graphene quantum point contact (blue curve). Theoretical results are rescaled to experimental device size as determined from panel a. Ideal transmission $\propto k_F$ is shown in red as guide to the eye. Curves are offset horizontally for clarity. **c**, Local density of states of graphene quantum point contact from tight-binding simulations, at three different energies (-100 meV, -30 meV and 250 meV; see also arrows in panel e). **d**, Graphene density of states extracted from experiment (fit to a Gaussian) and **e** from simulation. Both experiment and theory find a substantial contribution from trap states around the Dirac point.

extracted for each device and compared to its width W (Fig. 1f) determined from scanning electron microscopy (SEM) images (see, e.g., Fig. 1b). The estimates for c_0W extracted from $G^{(0)}$ lie just little below the width W , where c_0 decreases for decreasing width. This suggests that for the narrower devices reflections, most likely due to device geometry and edge roughness, are playing a more important role. From the data in Fig. 1f we can extract $c_0 \approx 0.56$ for our smallest constriction. Below we will show that, indeed, reflections at the rough edges of the constriction and not a reduction in active channel width is responsible for the deviation of the experimentally extracted c_0W from the SEM width W .

Localized states at the edges.

For small $k_F < 50 \times 10^6 \text{ m}^{-1}$ (i.e. low carrier concentrations) the measured conductances systematically deviate from the expected linear behavior (see Fig. 1e).

This deviation from the square-root relation between G and n (i.e. ΔV_g) becomes more apparent when focusing on G around the charge neutrality point (CNP). The conductance as function of n for two different cool-downs of the same graphene constriction ($W \approx 230$ nm, Fig. 2a), shows marked cool-down dependent low carrier density regions with substantial deviations from $G \propto \sqrt{n}$. Far away from the CNP, the conductance as function of n for both cool-downs shows (i) an identical \sqrt{n} behavior leading to the very same c_0W and (ii) almost identical, regularly spaced kink structures (see arrows in Fig. 2a), which are, however, slightly shifted relative to another on the carrier density axis n . These observations suggest that the square-root relation between the Fermi wave vector k_F and the gate voltage V_g , i.e. n needs to be modified. While the quantum capacitance of ideal graphene can be neglected^{26–28}, a small additional contribution $n_T(\Delta V_g)$ from, e.g., localized trap states modifies the relation between n and k_F to

$$\alpha \Delta V_g = n = k_F^2 \pi^{-1} + n_T(\Delta V_g). \quad (3)$$

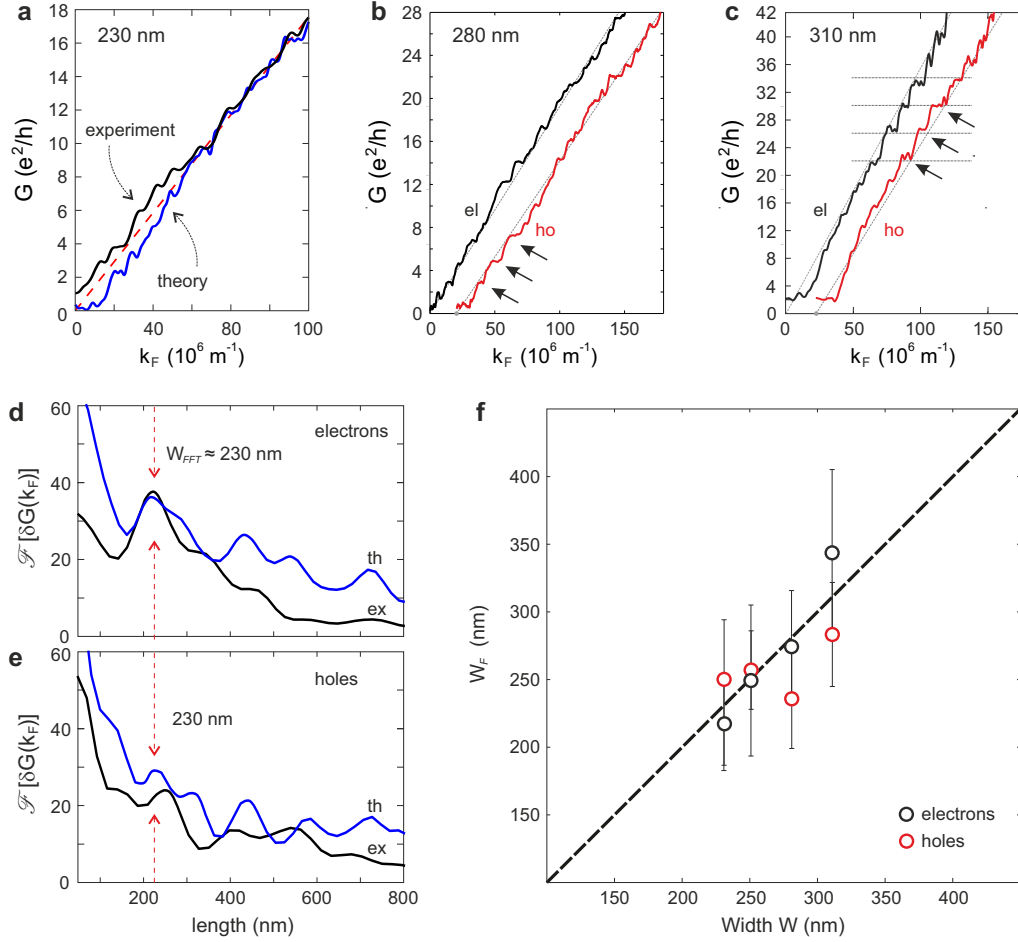


Figure 3. Size quantization signatures. **a**, Comparison of the low energy conductance between theory (blue) and experiment (black). **b**, **c**, Measured electron (el - black trace) and hole (ho - red trace) conductance including kink or step-like structure (see arrows) as a function of k_F for two different constriction geometries (see insets). The hole conductance traces are horizontally offset for clarity. **d**, Fourier transform of the $G - G^{(0)}$ electron conductance $\mathcal{F}[\delta G(k_F)]$ through the 230 nm graphene constriction, for experiment (ex - black trace) and theory (th - blue trace). The first peak of the Fourier transform clearly corresponds to the width W of the quantum point contact (marked by arrows). **e**, Same as **d** for the hole conductance. The size of the first peak is substantially reduced for both experiment and theory due to the presence of localized states that lead to additional scattering. **f**, Comparison of width W_F extracted from the Fourier transform of the conductance traces (as in panels e, f) to geometric constriction width W from four different devices (extracted from SEM images).

Far away from the Dirac point ($k_F^2 \gg \pi n_T$), we recover the expected square root relation. Close to the Dirac point, however, $\alpha \Delta V_g$ will be strongly modified by deviations n_T from the linear density of states of ideal Dirac fermions and approaches $n_T(\Delta V_g)$ near the CNP.

The trap states do not contribute to transport, yet they contribute to the charging characteristics³⁰. It is important to note that electron-hole puddles²⁹ or charged impurities would only smear out the density of states but would not add additional trap-state density n_T . This is in contrast to graphene edges, in particular rough graphene edges, which feature a significant number of trap states. For example, a tight-binding simulation of the local density of states of the experimental geometry yields a strong clustering of localized states at the device edges (see Fig. 2c), which energetically lie close to the

CNP (Fig. 2e). The deviation of G from the \sqrt{n} scaling also opens up the opportunity to extract n_T from experimental conductance data (e.g. Fig. 2d), and thus a new pathway for device characterization. Inspired by the tight-binding simulation, we approximate the trap state density n_T as function of Fermi wave vector by a Gaussian distribution. We fit the position, height and width of the Gaussian by minimizing the difference between the measured $G(k_F)$ and the corresponding linear extrapolation to very low values of k_F (see Fig. 2b and Supplementary Note 3). We find good qualitative agreement between simulation and experiment (compare Figs. 2d and 2e). Quantitative correspondence would require a detailed, microscopic model for the trap state density n_T . Note that the only difference between different traces in Figs. 2a, 2b and 2d is the exposition of

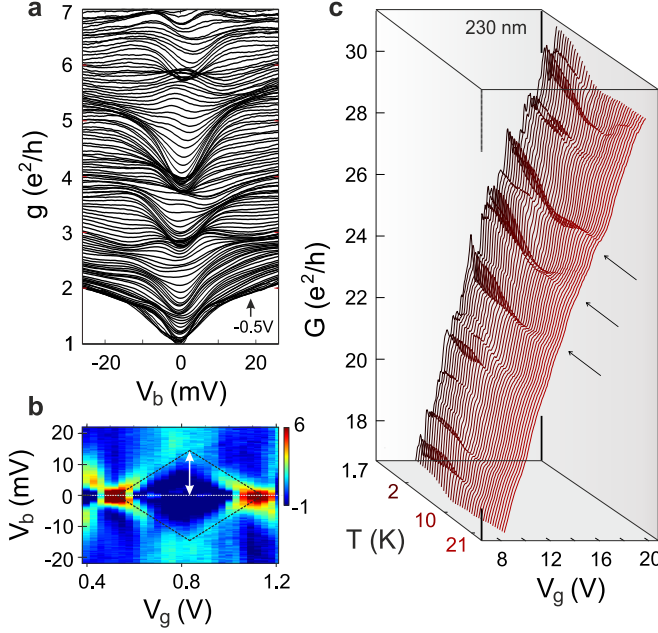


Figure 4. Quantized conductance: finite bias and temperature dependence. **a**, Zero B field differential conductance g as a function of bias voltage V_b , measured at $T = 6$ K, taken at fixed values of back-gate voltage V_g from -0.5 V to 3.0 V in steps of 30 mV (see lower right label). The dense regions correspond to plateaus in conductance. **b**, Transconductance $\partial g/\partial V_g$ in units of e^2/hV (see color-scale) as a function of bias and back gate voltage for a different cool-down of the same device (see also Supplementary Note 6). At $V_b = 0$, the transitions between conductance plateaus appear as red spots. At finite bias voltage, we observe a diamond like shape, which provides an energy scale for the subband energy spacing $\Delta E \approx 13.5 \pm 2$ meV (see dashed black lines and white arrow), which is also in good agreement with the energy scale observed in panel a (see also Supplementary Note 6). **c**, Conductance traces as a function of temperature and back gate voltage. We observe features with different temperature dependencies. Above around 10 K only kinks related to quantized conductance survive (see arrows).

the device to air for several days leading to a wider carrier density region of substantial deviations (green trace). The number of trap states (i.e., the deviations around the CNP) is significantly enhanced (compare also green and black trace in Fig. 2d). As the active graphene layer is completely sandwiched in hBN only the graphene edges are exposed to air and, very likely, experience chemical modifications. In line with our numerical results, we thus conjecture that localized states at the edges substantially contribute to n_T , leading to the strong cool-down dependence we observe in our measurements. While this interpretation seems plausible and is consistent with our data, alternative explanations cannot be ruled out.

Away from the CNP our data agrees remarkably well with ballistic transport simulations through the device geometry using a modular Green’s function approach⁸ (see blue trace in Fig. 2b): we simulate the 4-probe

constriction geometry taken from a SEM image, scaled down by a factor of four to obtain a numerically feasible problem size⁷. To account for the etched edges in the devices, we include an edge roughness amplitude of $\Delta W = 0.2W$ for the constriction. This comparatively large edge roughness (which is consistent with the systematic reduction of transmission through the constriction when using the average conductance) is probably due to microcracks at the edges of the device.

Quantized conductance.

Superimposed on the overall linear behavior of $G(k_F)$, we find reproducible modulations (“kinks”) in the conductance (see Figs. 3a-3c and Fig. S4b). The kinks are well reproduced for several cool downs (see arrows in Fig. 2a and Supplementary Note 4) as well as for different devices, generally showing a spacing ΔG varying in the range of $(2 - 4)e^2/h$ (see arrows in Figs. 3b and 3c). The “step height” and its sharpness depend on the carrier density (i.e. k_F) as well as on the constriction width and is strongly influenced by the overall transmission c_0 (Fig. 1f). Remarkably, we observe a spacing ΔG of the steps close to $4e^2/h$ for one of our wide samples ($W \approx 310$ nm) at elevated conductance values on both the electron and hole sides (see arrows and horizontal lines in Fig. 3c and Fig. S4b).

Our assignment of the conductance “kinks” as signatures of quantized flow through the constriction is supported by our theoretical results. Theory and experimental data from the smallest constriction show similar smoothed, irregular modulations (see Fig. 3a), instead of sharp size quantization steps.³³ The replacement of sharp quantization steps by kinks reflects the strong scattering at the rough edges of the device^{34,35}, resulting in the accumulation of random phases in the Fourier components of G [Eq. (2)]. We note that calculations with smaller edge disorder show a larger average conductance, yet very similar “kink” structures. As the present calculation includes only edge-disorder induced scattering while neglecting other scattering channels such as electron-electron or electron-phonon scattering, the good agreement with the data suggests edge scattering to be the dominant contribution to the formation of the “kinks”. By contrast, both experimental and theoretical investigations of, e.g., semiconducting GaAs heterostructures show very clear, pronounced quantization plateaus³⁶. In these heterostructures, the electron wave length near the Γ point is very long, and cannot resolve edge disorder on the nanometer scale. By contrast, K - K' scattering in graphene allows conduction electrons to probe disorder on a much shorter length scale. Consequently, edge roughness substantially impacts transport. The comparison between experimental and theoretical data (Fig. 3a) unambiguously establishes the observed modulations to be consistent with the smoothed size quantization effects predicted by theory.

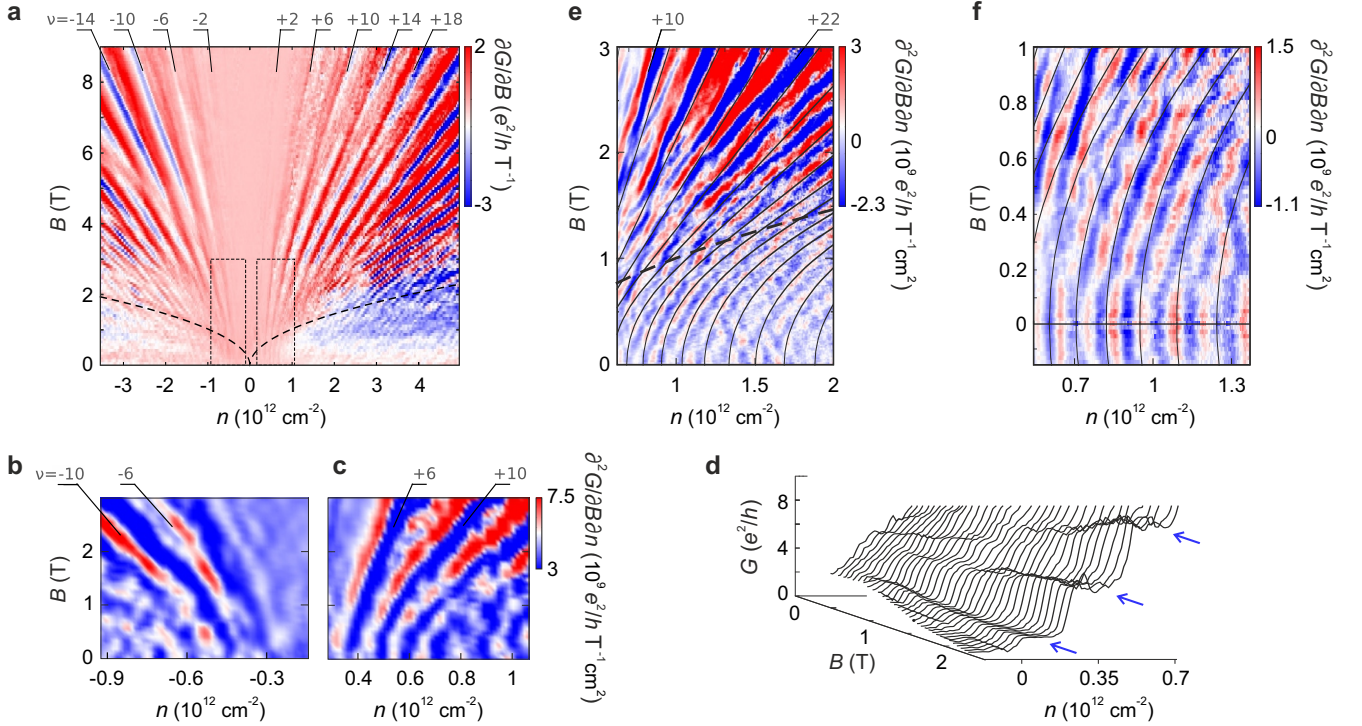


Figure 5. Magnetic field dependence of the size quantization. **a**, Landau level fan of the graphene quantum point contact of width $W = 230 \text{ nm}$, measured at $T = 1.7 \text{ K}$. Landau levels emerge at high magnetic fields. The magnetic field quantization of Landau level m dominates over size quantization as soon as $2\sqrt{2m}l_B$ (where the magnetic length $l_B \approx 25/\sqrt{B[\text{T}]}$ nm) is smaller than the constriction width (B field values above dashed black line). **b,c**, Double derivative plots of the regions delimited by thin dashed lines in panel **a** showing the evolution of the lowest quantization plateaus with magnetic field: we observe the full transition from quantized sub-bands ($B = 0 \text{ T}$) to Landau levels at large B field. **d**, The same magnetic field evolution is visible in the conductance as a function of magnetic field and charge carrier density for a different cool-down of the same device, also measured at 1.7 K . The blue arrows highlight the expected quantum Hall conductance plateaus at $2, 6$ and $10 e^2/h$. **e**, Double derivative plot of the conductance as a function of magnetic field and charge carrier density measured at $T = 6 \text{ K}$. The solid black lines denote the theoretical expectations for the evolution of the size quantization with magnetic field. The thick dashed black line corresponds to the boundary of the Landau level regime, also appearing in panel **a**. **f**, Zoom-in of panel **e** for small magnetic fields $B \leq 1 \text{ T}$.

By subtracting the zeroth-order Fourier component $\propto k_F$ (or \sqrt{n}), the superimposed modulations of the conductance $\delta G(k_F) = G - G^{(0)}$ provide direct information on the quantized conductance through the constriction [Eq. (2)]. One key observation is that the Fourier transform of $\delta G(k_F)$ offers an alternative route towards the determination of the constriction width complementary to that from the mean conductance $G^{(0)}$. For example, the pronounced peak of the first harmonic at 230 nm (red arrows in Figs. 3d and 3e) is consistent with the constriction width W derived from the SEM image. Interestingly, our simulation also correctly reproduces the experimental observation that the peak in the Fourier spectrum of $\delta G(k_F)$ is more pronounced on the electron side (Fig. 3d) than on the hole side. This results from the slightly asymmetric energy distribution of the trap states relative to the CNP, which is accounted for in our tight-binding calculation.

Performing such a Fourier analysis for several devices (Supplementary Note 5) yields much closer agreement

with the geometric width W (Fig. 3f and horizontal axis of Fig. 1f) than an estimate based only on the zeroth-order Fourier component $c_0 W$ [first term in Eq. (2), see vertical axis of Fig. 1f]. Fourier spectroscopy of conductance modulations thus allows to disentangle reduced transmission due to scattering at the edges ($\tilde{c}_0 W$) from the effective width of the constriction, and proves the relation between the observed Fourier periodicity and the device geometry.

Bias voltage spectroscopy measurements yield an estimate for the energy scale of the size quantization steps^{11,37}. For example, by analyzing finite bias measurements from our smallest constriction device we extract a subband energy spacing of $\Delta E = 13.5 \pm 2 \text{ meV}$ near the CNP (Figs. 4a, 4b and Supplementary Note 6). With the geometric width of 230 nm also confirmed by the Fourier spectroscopy (Fig. 3c) we can estimate the Fermi velocity near the CNP as $v_F = 2W\Delta E/\hbar = (1.5 \pm 0.2) \times 10^6 \text{ m/s}$. This is a clear signature of a substantially renormalized Fermi velocity in nanostructured graphene, possibly

enhanced by electron-electron interaction³⁸. Moreover, the extracted energy scales are consistent with the weak temperature dependence of the quantized conductance (Fig. 4c and Supplementary Note 7).

Transition from quantized conductance to quantum Hall.

Additional clear fingerprints of size quantization appear in the parametric evolution of the conductance steps³⁹ with magnetic field, B . The transition from size quantization at zero B -field to Landau quantization at high magnetic fields occurs when the cyclotron radius l_C becomes smaller than half the constriction width W . For the Landau level m the transition should occur at $2l_C = 2\sqrt{2m}l_B \approx W$ with l_B the magnetic length. This transition line in the $B-n$ plane (see black dashed curve in Fig. 5a) agrees well with the onset of Landau level formation in our data (see Supplementary Note 8 for similar data from a 280 nm constriction device). The evolution of the lowest quantized steps (at $B = 0$ T) to the corresponding lowest Landau levels at low temperatures ($T=1.7$ K) can be easily tracked (see Figs. 5b and 5c). At higher temperatures ($T = 6$ K) the evolution of quantized sub-bands to Landau levels is observed even for higher conductance plateaus (Fig. 5d, 5e). For a comparison, we calculate the evolution of size quantization of an infinitely long ribbon of width W as function of magnetic field. We take $W \approx 230$ nm from the SEM data, which leaves no adjustable parameters. Our model (black lines in Figs. 5e and 5f) reproduces the evolution from the kinks at small fields ($l_B \gg W$) to the Landau levels for large fields ($l_B < W$) remarkably well, further supporting the notion that they are, indeed, signature of size quantization.

DISCUSSION

We have shown ballistic conductance of confined Dirac fermions in high-mobility graphene nanoconstrictions

sandwiched by hexagonal boron nitride. Away from the Dirac point, we observe a linear increase in conductance as function of Fermi wavevector with a slope proportional to constriction width. Close to the Dirac point, the charging of localized edge states distorts this linear relation. Superimposed on the linear conductance, we observe reproducible, evenly spaced modulations (“kinks”). Tight-binding simulations for the device reproduce these structures related to size quantization at the constriction. We can unambiguously identify these “kinks” as size quantization signatures by both Fourier spectroscopy at zero magnetic field and their evolution with magnetic field, finding good agreement between theory and experiment.

METHODS

Experimental methods and details

The hBN-graphene-hBN sandwich structures⁸ have been etched by reactive ion etching in a SF_6 atmosphere, prior deposition of a ~ 10 nm-thick Cr etching mask. Remaining rests of Cr oxide are removed by immersing the samples in a Tetramethylammonium hydroxide (TMAH) solution for about 30-35 s. All transport measurements are performed in a 4-probe configuration using standard lock-in techniques. Since the distances between the contacted current-carrying electrodes and the voltage probes are small, compared to the other length scales of the system, we have an effective 2-probe configuration. Importantly, this way we exclude the one-dimensional contact resistances.

Electrostatic simulations and transport calculations

We simulate the experimental device geometry using a third-nearest neighbor tight-binding ansatz. We rescale our device by a factor of four compared to experiment, to arrive at a numerically feasible geometry. We determine the Green’s function using the modular recursive Green’s function method^{8,9}. The local density of states and transport properties can then be extracted by suitable projections on the Green’s function. For more technical details see Supplementary Note 9.

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Author contributions

B.T. and C.S. conceived the project. B.T. fabricated the samples, performed the experiments and interpreted the data. S.E. assisted during measurements. B.T. and D.J analyzed the data. L.A.C. and F.L. performed the numerical calculations and theoretical analysis. A.G. and F.L. developed the numerical code. T.T. and K.W. synthesized the hBN crystals. J.B., S.V.R. and C.S. advised on theory and experiments. B.T., L.A.C, F.L., J.B., and C.S. prepared the manuscript. All authors contributed in discussions and writing of the manuscript.

Competing financial interests

The authors declare no competing financial interests.

SUPPLEMENTARY INFORMATION

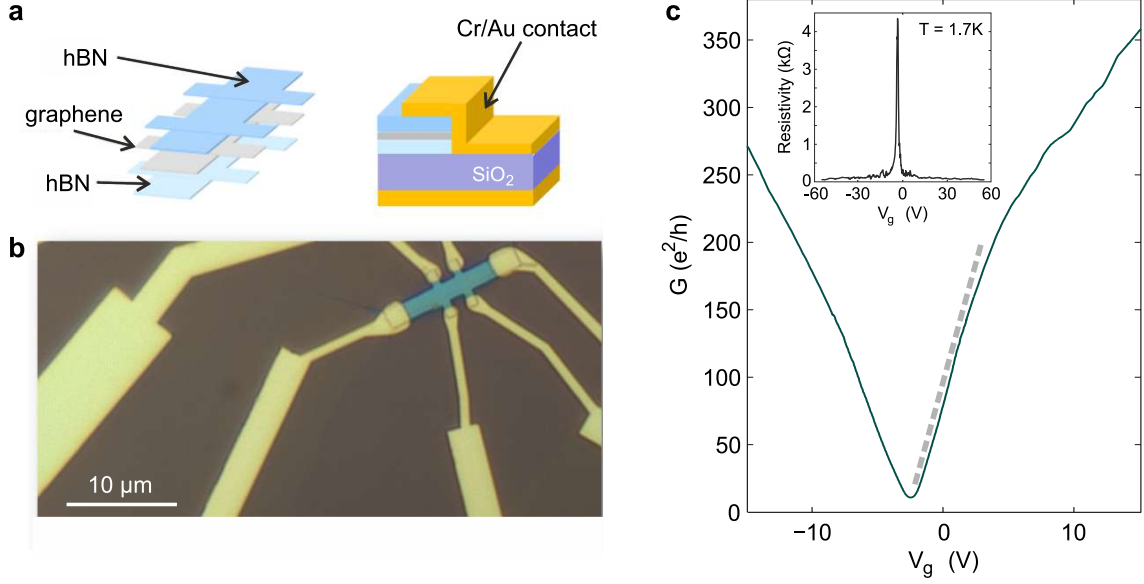
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In this Supplementary Information we provide additional experimental data as well as a detailed description of the experimental and theoretical methods expanding the ‘Methods’ section of the main article. Within the Supplementary Information, references are numbered as, e.g., equation (S1) and Figure S1, whereas regular numbers, e.g., equation (1) and Figure 1, refer to the main article.

Supplementary Note 1. Sample quality

The field-effect carrier mobility in our sandwich devices is on the order of $150.000 \text{ cm}^2/\text{Vs}$. This high sample quality is thanks to advances in sample fabrication, in particular the van-der-Waals stacking process: the graphene is fully encapsulated in hBN, resulting in significantly improved sample quality. We extract the mobility from a one μm -wide Hall bar device fabricated in the very same batch as our graphene constrictions (see Fig. S1). The dark blue trace in Fig. 1d of the main manuscript is taken from this Hall bar device. As all traces from the constrictions with different widths (some of them carved out from the same hBN-graphene-hBN sandwich) lie systematically below the Hall bar trace, we exclude bulk scattering as limiting process in our devices. Independently, we have shown recently in a collaboration with A. Morpurgo, F. Guinea and coworkers¹ that in our high-quality devices the carrier mobility is not limited by charge impurity and short-range scattering but rather by nanometer-scale strain variations giving rise to long-range scattering with allowed pseudospin flips. We expect that the same limitations on the mean free path also apply to our graphene constriction devices.



Supplementary Figure S1. Reference hBN-graphene-hBN sandwich Hall bar device. (a) Schematic illustration of the hBN-graphene-hBN sandwich structure and nature of the quasi-one-dimensional graphene-metal (Cr/Au) contact. (b) Optical image of an etched and contacted $\sim 1 \mu\text{m}$ -wide hBN-graphene-hBN sandwich Hall bar device. (c) Four-terminal conductance as a function of back gate voltage V_g measured at a constant current of 50 nA and a temperature of 16 K. From the linear slope near the charge neutrality point (see dashed line) we extract a carrier mobility of around $150.000 \text{ cm}^2/\text{Vs}$. The inset shows the four-terminal resistivity as a function of gate voltage at lower temperature (1.7 K).

SEM width W (nm)	α ($10^{10}/\text{cm}^2\text{V}$)
1000	7.00
850	5.80
590	6.75
440	6.90
310	7.00
280	7.20
250	5.40
230	7.15

Supplementary Table S1. Lever arm values α for eight different devices extracted from the Landau level fan measurements (see Fig. S2).

Supplementary Note 2. Extraction of the gate lever arm α

Measurements of Landau levels in graphene as a function of back gate voltage V_g and magnetic field B (see Fig. S2) allow for an independent determination of the gate coupling (or lever arm) α . The Landau level spectrum for massless Dirac fermions in graphene is given by

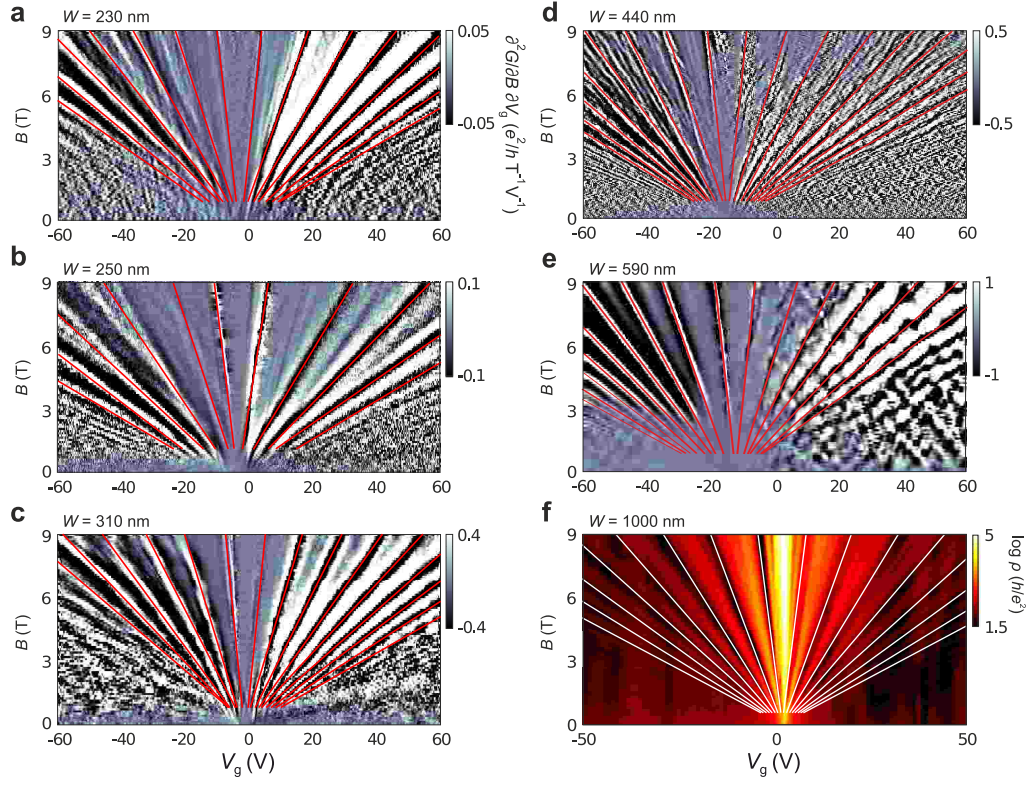
$$E_m(B) = \text{sgn}(m)v_F\sqrt{2|e|\hbar|m|B}, \quad m \in \mathbb{Z}_0, \quad (\text{S1})$$

where v_F is the Fermi velocity and m is the quantum number of the corresponding Landau level. Assuming a perfect linear dispersion and a constant capacitive gate coupling leads to the following relation between energy E and back gate voltage

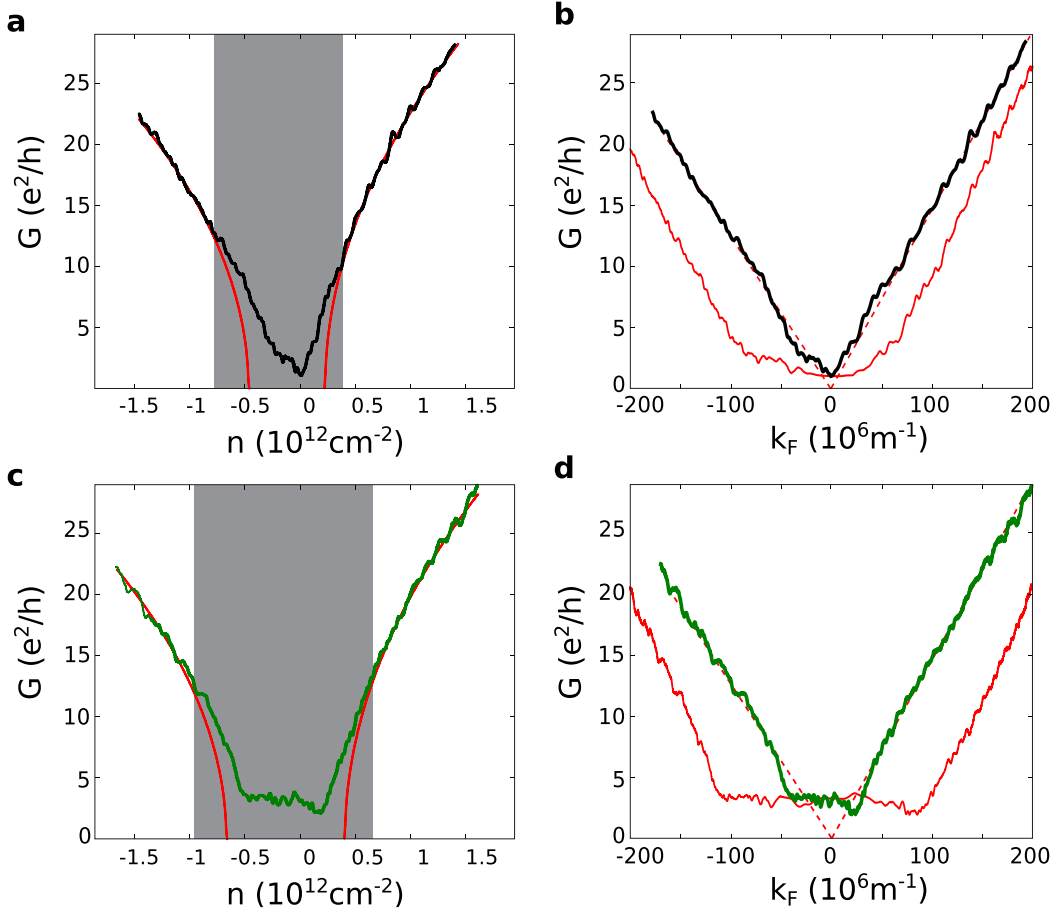
$$E = \hbar v_F k_F = \hbar v_F \sqrt{\pi \alpha \Delta V_g}, \quad (\text{S2})$$

where $\Delta V_g = V_g - V_g^0$, and V_g^0 is the gate voltage at the charge neutrality point. As a result, the Landau levels in the $B - V_g$ plane form straight lines, i.e. $B_m = C_m \Delta V_g$, where the slope $C_m = \alpha \hbar / 4 m e$ is Landau level index (m) dependent and proportional to the capacitive coupling α (see red lines in Fig. S2a-e).

The onset of each Landau level can be resolved by taking the mixed second derivative of the longitudinal conductance G with respect to V_g and B , i.e. $\partial^2 G / \partial V_g \partial B$. The positions of the Landau levels coincide with the minima/maxima of the derivative on the electron/hole side (see Fig. S2a-e, where the local minima/maxima coincide with red lines). Alternatively, the Landau levels can be determined from the minima of the longitudinal resistivity ρ (marked in white in Fig. S2f). Note that C_m is independent of the Fermi velocity, experimental determination of which is rather difficult. Table S1 summarizes the extracted values of α for the different devices.



Supplementary Figure S2. Landau fan and capacitive coupling. (a)-(e) Second derivative of the longitudinal conductance $\partial^2 G / \partial V_g \partial B$ as a function of magnetic field B and back-gate voltage V_g for six different devices with different widths. The red lines follow the evolution of the Landau levels. The slopes of the lines are proportional to the capacitive coupling α . (f) The longitudinal resistivity ρ as a function of B and V_g provide an alternative way to extract α from the position of the Landau levels, marked by white lines.



Supplementary Figure S3. Deviation from the ideal ballistic conductance for the 230 nm-wide graphene constrictions. (a) Low-bias four-terminal conductance G as a function of charge carrier density n . The red solid lines are fits to a simple capacitive coupling model [Eq. (S2)] valid at high carrier densities for the holes and electrons regime, respectively. Deviations appear in the gray-shaded region around the charge neutrality point. (b) Conductance G of panel (a) as a function of k_F using the linear density of states of ideal graphene (red solid line), or including a finite density of trap states (Eq. S4) around the Dirac point (black solid line). The linear relation between $G(k_F)$ and k_F expected for ideal graphene is shown as a dashed red line. (c) and (d) Corresponding to (a) and (b) but for a different cool-down of the same constriction. After exposing the sample to ambient conditions, the number of charge traps responsible for the flat area around the Dirac point increased significantly.

Supplementary Note 3. Linearization of G as a function of k_F

For a known gate coupling α , one can evaluate the measured conductance $G(V_g)$ as a function of k_F , using the standard constant capacitive coupling model $k_F = \sqrt{\pi\alpha\Delta V_g}$. Following the Landauer theory of conductance through a constriction of finite width W , the averaged conductance $G^{(0)}(V_g)$ features a square-root dependence on V_g ,

$$G^{(0)} = \frac{4e^2}{h} \left(\frac{c_0 W k_F}{\pi} - \frac{c_0}{2} \right) = \frac{4e^2}{h} \frac{c_0 W}{\pi} \sqrt{\pi\alpha(V_g - V_g^0)} - \frac{2e^2}{h} c_0. \quad (\text{S3})$$

A closer look at the traces from two different cool-downs of the narrowest device with $W = 230$ nm (Figs. S3a and S3c) reveals a systematical deviation from the expected square-root dependence of G [Eq. (S3)] at low carrier concentrations, i.e for $n < 0.45 \times 10^{12} \text{ cm}^{-2}$ on the electron side and $n < 0.75 \times 10^{12} \text{ cm}^{-2}$ on the hole side (Fig. S3a). This deviation becomes more pronounced closer to the charge neutrality point (see shaded area in Figs. S3a and S3c). In the ballistic region, i.e., far from the charge neutrality point, we can use Eq. (S3), with α extracted from the Landau level fan, and fit parameters $V_g^{0,e}$ for the electron (e) and $V_g^{0,h}$ for the hole (h) side. As expected, the conductance G evolves linearly as function of k_F in the ballistic regime (see red traces in Figs. S3b and S3d), but large deviations between data and model become apparent close to the charge neutrality point. We conclude that a linear model using

a constant gate coupling is not directly applicable to our graphene constriction devices. Instead, one needs to account for the additional charge carrier trap states n_T (see main text), modifying the relation between back-gate voltage and Fermi wave number according to

$$\alpha(V_g - V_g^0) = \alpha\Delta V_g = k_F^2\pi^{-1} + n_T(\Delta V_g). \quad (\text{S4})$$

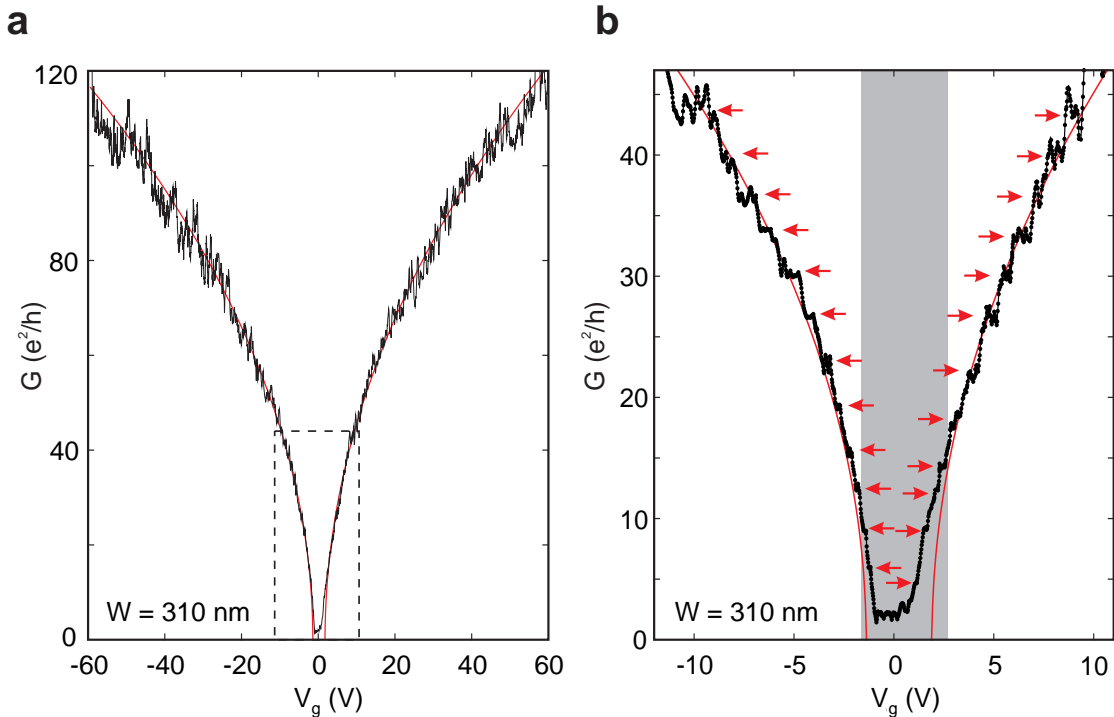
Using Eq. S4, we obtain an implicit mapping $k_F(\Delta V_g)$, which depends on the functional form of $n_T(\Delta V_g)$ and accounts for the modified density of states in the constriction,

$$k_F(\Delta V_g) = \sqrt{\pi\alpha\Delta V_g - \pi n_T(\Delta V_g)}. \quad (\text{S5})$$

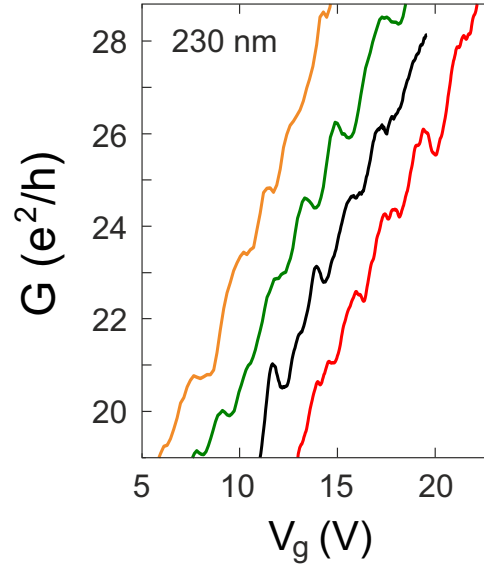
We conjecture that the strong cool-down dependence seen in the different traces of Fig. S3 are due to modifications in the trap state densities as the sample was exposed to air². As the graphene layer in our hBN-graphene-hBN sandwich can only interact with air at the edges, edge states presumably strongly contribute to n_T . Indeed, tight-binding simulations of the constriction geometry (see main text, Fig. 2c,e) yield a clustering of localized edge states close to the Dirac point. Accounting for n_T by Eq. (S5) should recover the linear relation between Fermi wave number and conductance. We can thus determine n_T from the measured conductance: we assume a Gaussian distribution of trap states n_T , and fit the width, position and height of the Gaussian distribution by minimizing deviations of the rescaled conductance $G[(k_F(\Delta V_g)]$ from the linear conductance $G^{(0)}(k_F)$ of Eq. (S3), see green/black traces in Figs. S3b and S3d. Note that this procedure assumes that any other sources for a deviation from a linear relation between k_F and G (due to, e.g., many-body effects) are small compared to the contribution from trap states n_T .

Supplementary Note 4. Reproducibility of kink signatures

We find regular kink structures in the conductance trace of our constriction devices (see, e.g., arrows in Fig. S4b). These kinks are well reproducible for different cool-downs of the same device (see Figs. S5, S6), and appear in



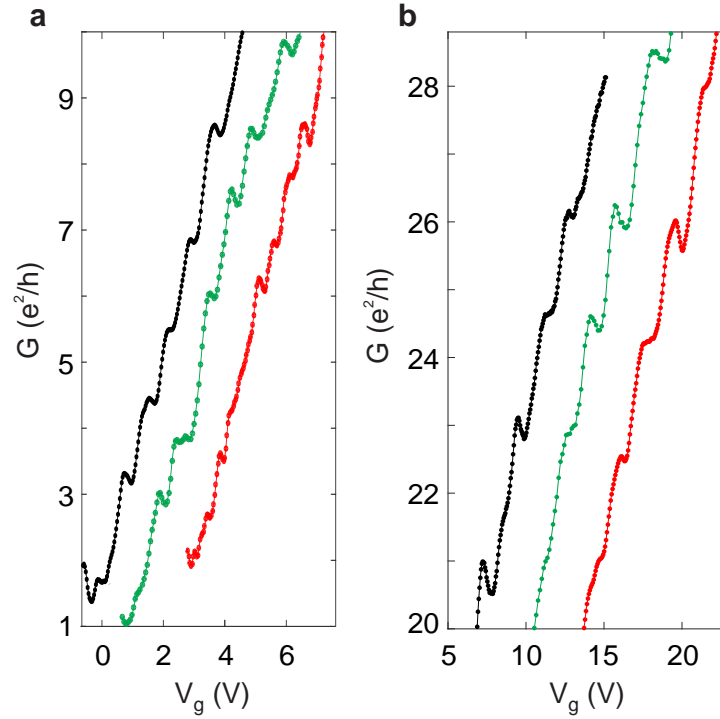
Supplementary Figure S4. Kinks in the back-gate characteristics of the 310 nm-wide graphene constriction (a) Low-bias four-terminal conductance G as a function of back gate voltage V_g , measured at $T = 2$ K. The ideal Landau-Büttiker model of conductance $G \propto \sqrt{n}$ is marked in red. **(b)** Close-up of the conductance G inside the dashed-line region of panel a. The reproducible kinks are clearly visible (marked by red arrows). The shaded gray region denote deviations from the ideal Landauer model (red trace).



Supplementary Figure S5. Cool-down dependence of the kinks for the 230 nm-wide graphene constriction I. Four-terminal conductance G as a function of back gate voltage V_g for four different cool-downs of the 230 nm-wide graphene constriction. The traces are shifted horizontally for clarity.

conductance data of several different devices (see Fig. S7). Analyzing the position of kinks as a function of back-gate voltage offers an independent check of the trap state density n_T .

In a first order approximation, the band structure of a graphene constriction of width W can be described as a collection of one-dimensional subbands originating from the quantization of the wave vector perpendicular to the

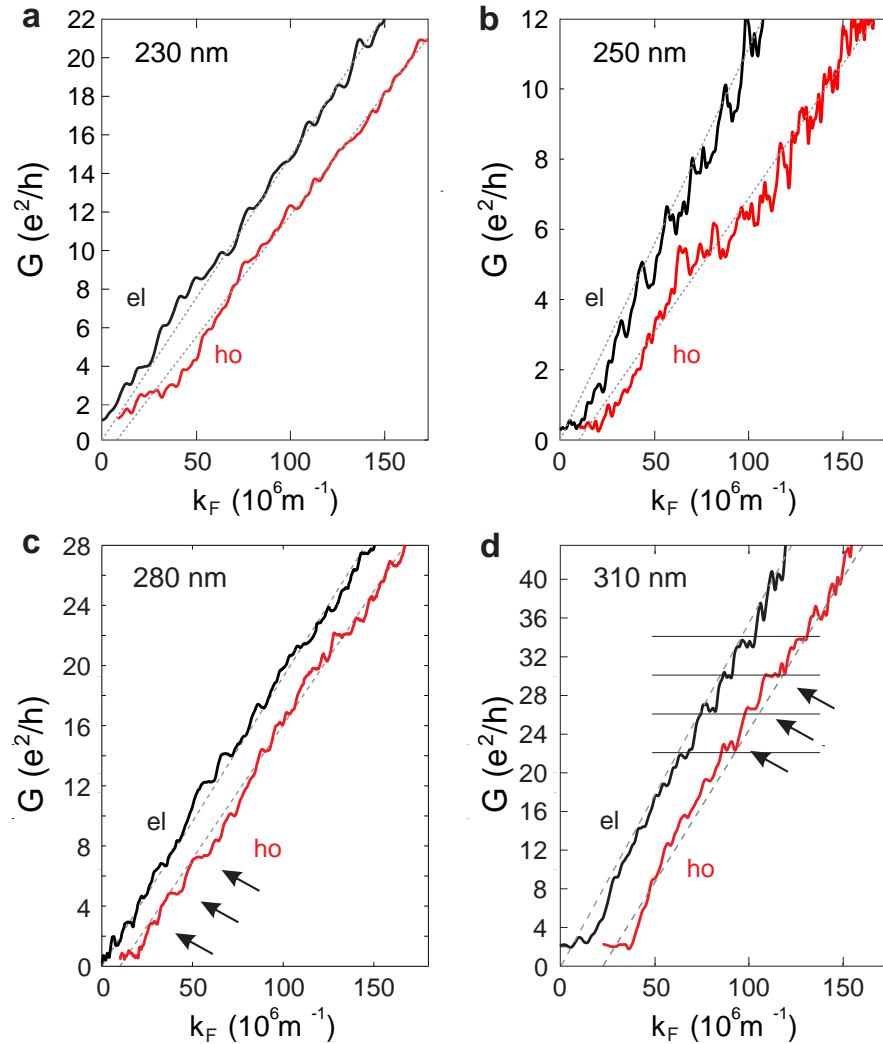


Supplementary Figure S6. Cool-down dependence of the kinks for the 230 nm-wide graphene constriction II. (a) and (b) Four-terminal conductance G as a function of back gate voltage V_g for different cool-downs of the 230 nm-wide graphene constriction at low and high charge carrier densities (panels a and b, respectively). The traces are shifted horizontally for clarity.

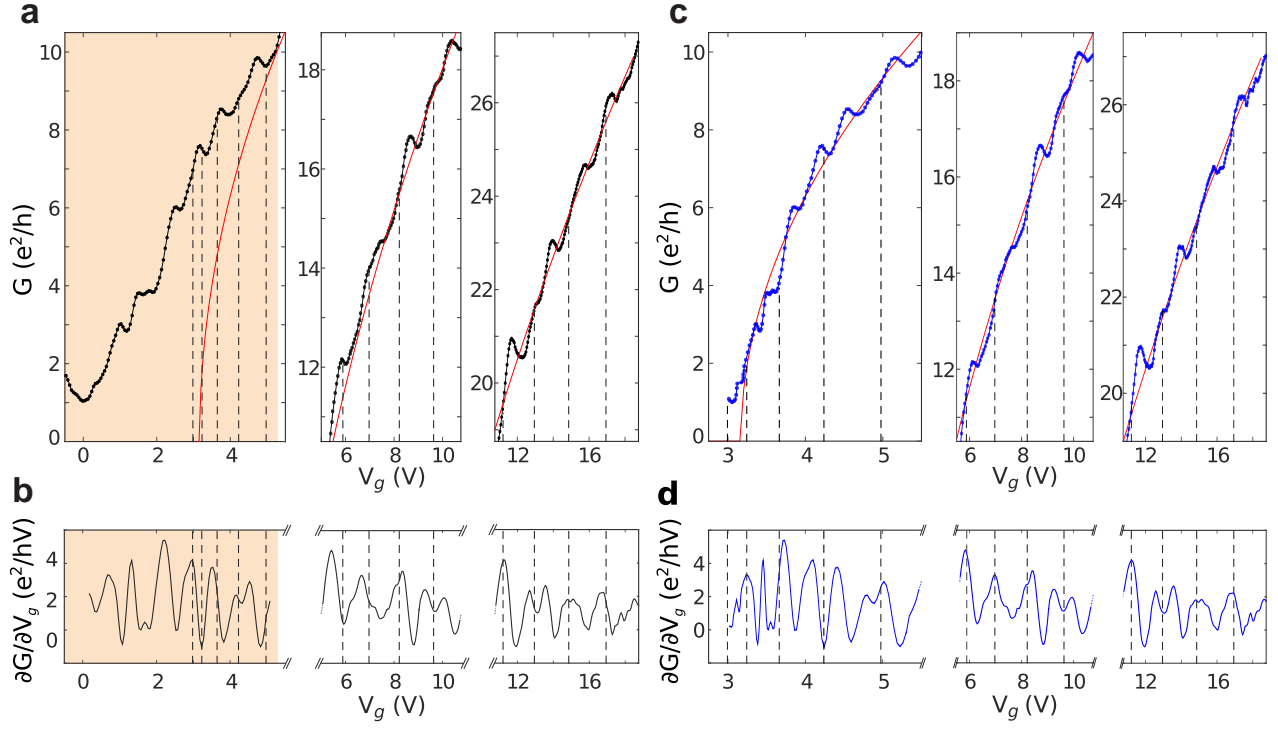
transport direction,

$$k_{\perp} = \pm |M + \beta| \pi / W, \quad (\text{S6})$$

where $M = 0, \pm 1, \pm 2, \dots$ is an integer associated with the subband index (both signs emerge due to the presence of two cones), and $0 \leq |\beta| < 0.5$ is a Maslov index related to the boundary conditions at the edges (for simplicity we use $\beta = 0$, i.e. a zigzag ribbon). Within the energy range where the ballistic model (see red trace in Fig. S8) fits the conductance trace, the theoretical position of the subbands (marked by vertical black dashed lines in Fig. S8) for a 230 nm-wide graphene constriction ($V_g^M = \pi M^2 / \alpha W^2$, $M = 1, 2, \dots$) are in good agreement with the kinks in the conductance (see Fig. S8a). The agreement between model and data is also visible in the derivative of the conductance $\partial G / \partial V_g$ (see Fig. S8b). Close to the charge neutrality point though, the kink signatures do not appear to follow the theoretical position of the subbands (vertical black dashed lines in Fig. S8a,b). Upon rescaling k_F according to Eq. (S5) (independently determined from the average transmission), the kinks are shifted, in good agreement with the quantization model (see comparison between dashed vertical lines and the position of the kinks in Fig. S8c,d). In summary, we find that the rescaling according to Eq. (S5) will (i) realign similar, reproducible kink-structures of different cool-downs on the k_F axis and (ii) shifts the kink positions to fit the simple quantization model of Eq. (S6).



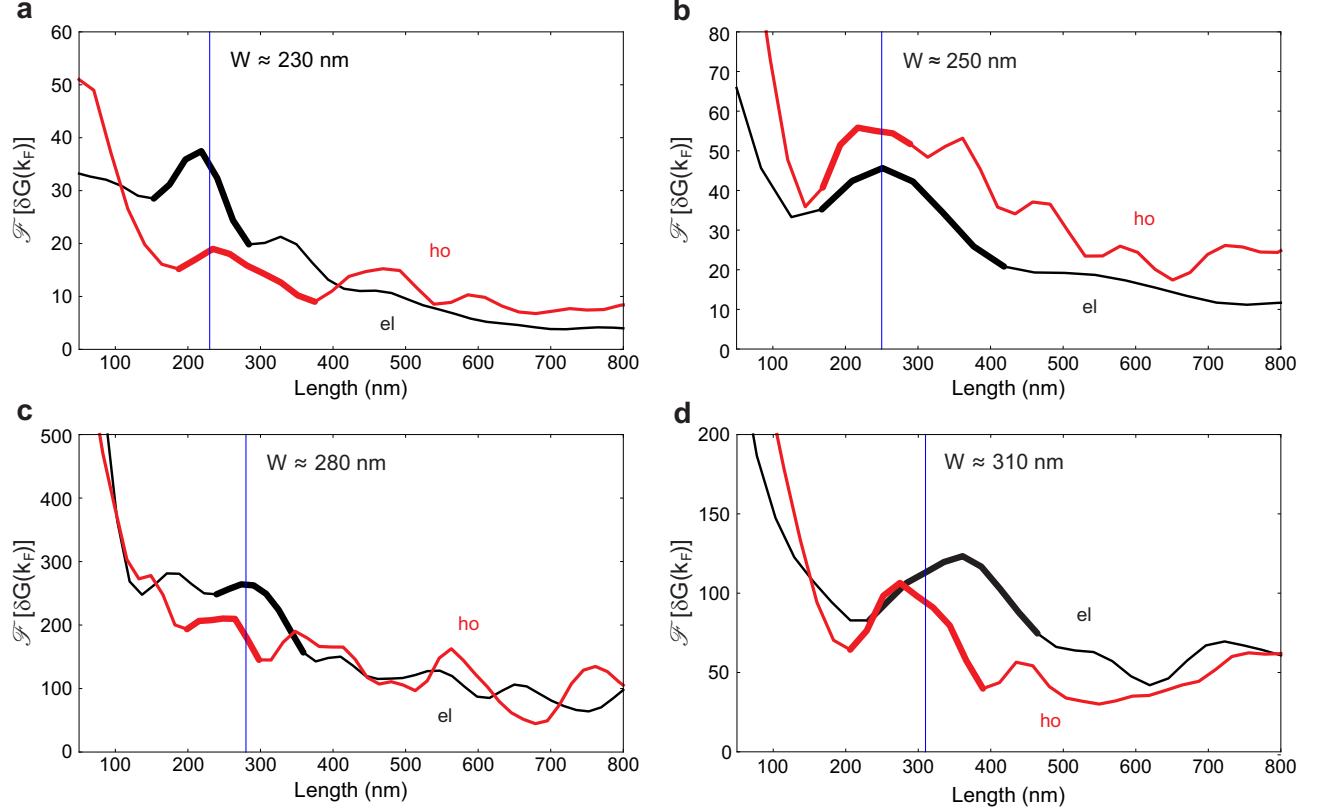
Supplementary Figure S7. Width dependence of the kinks in conductance. Four-terminal conductance G as a function of back gate voltage V_g for four different devices of widths 230 nm (a), 250 nm (b), 280 nm (c) and 310 nm (d). The transmission traces are shown in black (red) for electrons (holes) as a function of rescaled k_F (see main text). The arrows point to kinks where the conductance jumps by about $c_0 \times \frac{4e^2}{h}$, with c_0 as measure for the overall transmission of the device, see Eq. (S3); $c_0 \approx 0.95$ for the 310 nm constriction. The traces are shifted horizontally for clarity.



Supplementary Figure S8. Back-gate characteristics of the energy subbands of the 230 nm-wide graphene constriction. (a) Low-bias four-terminal conductance G as a function of back-gate voltage V_g . The theoretical position of the subbands in the V_g -axis is indicated by vertical dashed lines. Close to the Dirac point (leftmost subpanel) measurements deviate from the ideal Landau model $G \propto \sqrt{V_g}$ shown in red (orange-shaded region). (b) Derivative plot $\partial G / \partial V_g$ of the conductance trace shown in panel (a). The correlation between the expected position of the subbands (vertical dashed lines) and measurements holds only at high carrier densities. (c) Same as (a) after rescaling of the charge carrier density (Eq. 3). The vertical dashed lines indicating the theoretical position of the subbands matches now the positions of kinks. (d) Derivative plot $\partial G / \partial V_g$ of the conductance trace in panel (b).

Supplementary Note 5. Fourier spectroscopy of transmission data

Once the conductance is represented as a function of k_F , the Fourier transform of $\delta G(k_F)$ offers alternative information on the quantized conductance through the constriction. If the regular kinks we identify in our conductance data, indeed, correspond to size quantization signatures, we can extract the constriction width from the first peak of the Fourier transform. Comparison between the first peak in the Fourier transform of the measured conductance $G(k_F) - G^{(0)}(k_F)$ of four constriction devices (see Fig. S9 and Fig. 4 in the main text) to the geometric width W of the constriction, yields good agreement (see also Fig. 3f of the main text).



Supplementary Figure S9. Fourier transform of the conductance. Fourier transform of the electron (black) and hole (red) conductance for the devices of width 230 nm (a), 250 nm (b), 280 nm (c) and 310 nm (d). The widths extracted from the Fourier analysis W_F (peaks in $\mathcal{F}[\delta G(k_F)]$) are in good agreement with the widths extracted from SEM images (blue vertical lines). The extracted widths W_F and associated errors bars are shown in Fig. 3f.

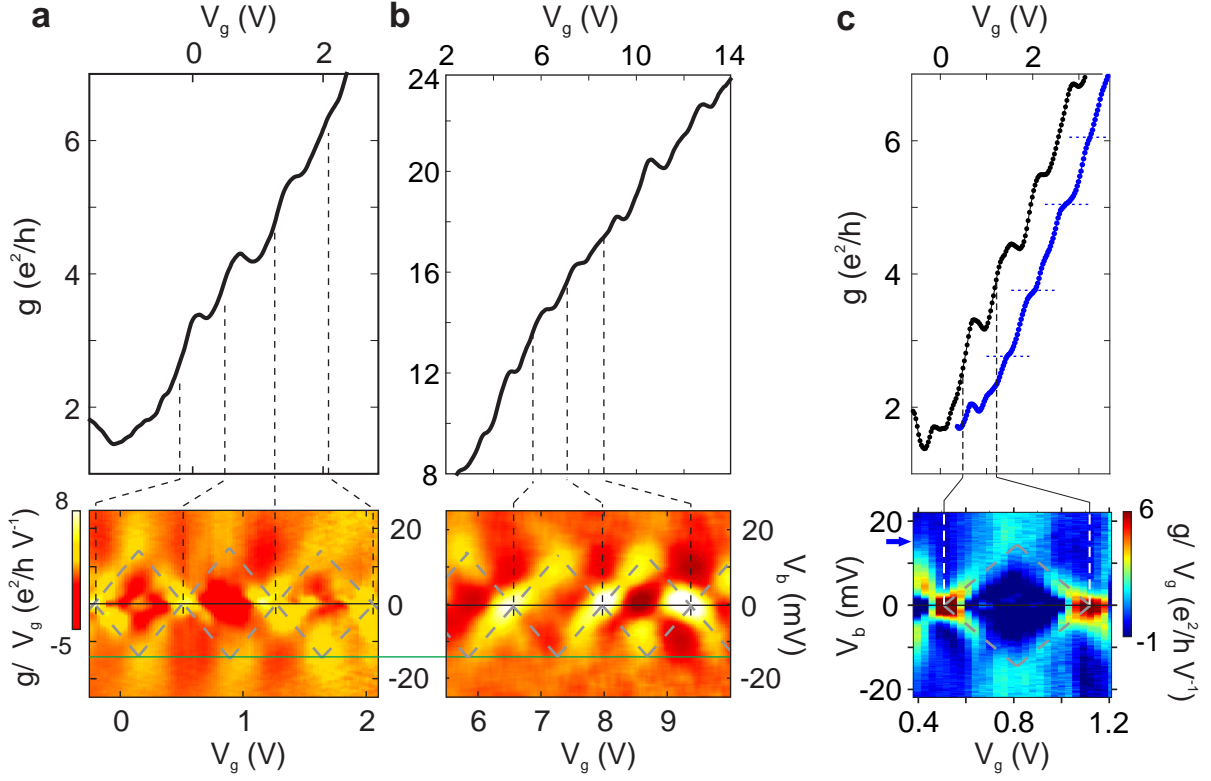
Supplementary Note 6. Bias spectroscopy

Using bias spectroscopy we can extract the energy scale associated with the regular kink pattern. The differential conductance $g = dI/dV = I_{SD}/V_{SD}$ (Fig. 4, Fig. S10 and Fig. S11) is measured from an AC excitation voltage $V_{AC} = 250 \mu V_{PP}$, using standard Lock-In techniques. We analyze six diamonds associated with kinks at the low- and high-conductance ranges (see Fig. S10). Extraction of the energy scale from the derivative of the differential conductance (color panels) yields $\Delta E = 13.5 \pm 2$ meV leading to $v_F = (1.5 \pm 0.2) \times 10^6$ m/s. Variations in the data are due to temperature effects, potential variations and uncertainties in determining the exact extensions of the diamonds. All six extracted diamonds are taken from energy regions where size quantization signatures are clearly visible and reproducible - we are thus confident that the sample is in the quantum point contact regime for all six diamonds. Note that modifications of the gate-lever arm do not affect the bias spectroscopy data since all energy scales are extracted from the bias voltage axis (V_b), which represents a direct energy-scale.

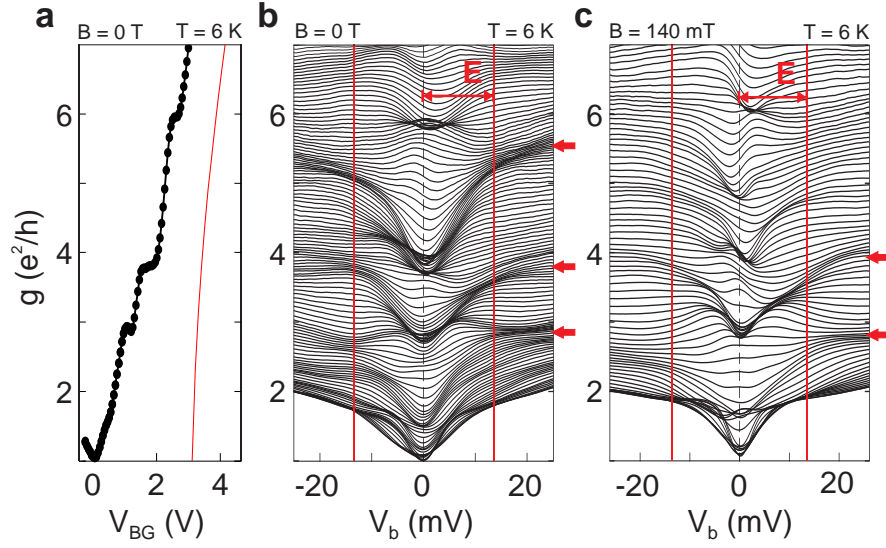
We extract similar values of subband spacing ($\Delta E \approx 13.5 \pm 2$ and 13.5 ± 3 meV) in a second (Fig. 4b of the main text and Fig. S10c) and a third (Fig. 4a of the main text and Fig. S11) cool-down of the same device. The value of subband spacing is additionally confirmed at finite magnetic field (Fig. S11c). We note that, at $B = 140$ mT, the quantized subbands are still caused by geometric confinement rather than magnetic confinement (i.e., due to the quantum Hall effect).

Moreover, half-conductance kinks^{3,4} are expected to emerge for a bias window eV_b greater than the subband spacing. Indeed, additional kinks at intermediate values of conductance are observed (horizontal dashed blue lines in Fig. S10c and red arrows in Fig. S11b,c). The observation of these intermediate kinks confirms the confinement nature of the observed kinks in conductance^{3,5,6}.

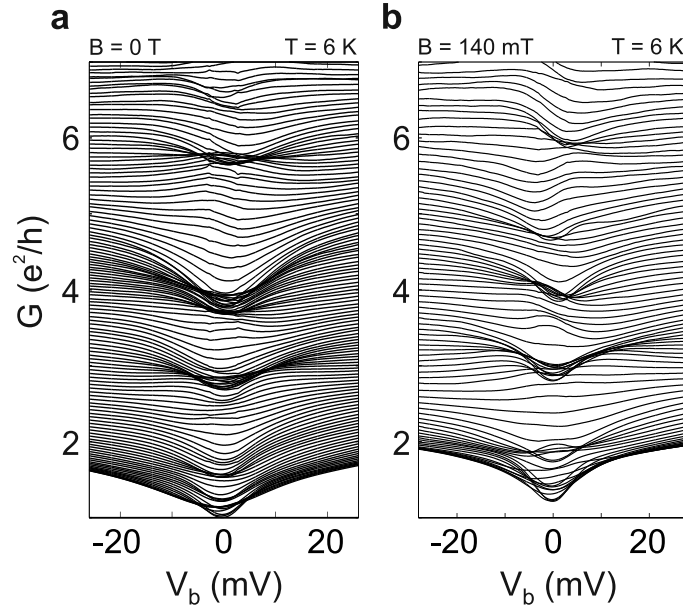
To check against any spurious contribution from the AC measurement technique, the bias spectroscopy measurements have been repeated in a DC configuration (Fig. S12). The conductance $G = I/V = I^{DC}/V_b$ is obtained from a symmetrically applied source-drain DC bias voltage V_b . Although the resolution of the DC conductance G (Fig. S12) is not sufficient to extract the subband spacing ΔE , the conductance kinks are still visible at identical values of conductance as in the AC measurements (Fig. S11).



Supplementary Figure S10. Bias spectroscopy of the 230 nm-wide graphene constriction. (a) Differential conductance g (upper panel) and differential transconductance $\partial g / \partial V_g$ (lower panel) as a function of back gate V_g and bias V_b voltages, measured at $B = 0$ T and $T = 6$ K. The differential conductance g (top panel) is measured at $V_b = 0$ V in the low carrier density range. The vertical black dashed lines indicate the position of the analyzed subbands. The transconductance $\partial g / \partial V_g$ (bottom color-scaled panel), of the data shown in the upper panel, is measured as a function of an applied bias voltage V_b . The kinks are characterized by high values (yellow color) of transconductance. The diamond structures are highlighted by dashed gray diamonds. We extract an average subband spacing $\Delta E \approx 13.5 \pm 2$ meV (green line). (b) Same as panel (a) measured at high carrier densities. (c) Same as panel (a) for a second cool-down of the same device. The blue trace represents the differential conductance g measured at $V_b = 15$ mV (see blue arrow in lower colored panel). The horizontal dashed blue lines highlight the levels of conductance of the intermediate kinks, visible (blue conductance trace) for energies above the subband spacing, e.g. $E \approx 15$ meV $>$ ΔE (blue arrow in lower colored panel).



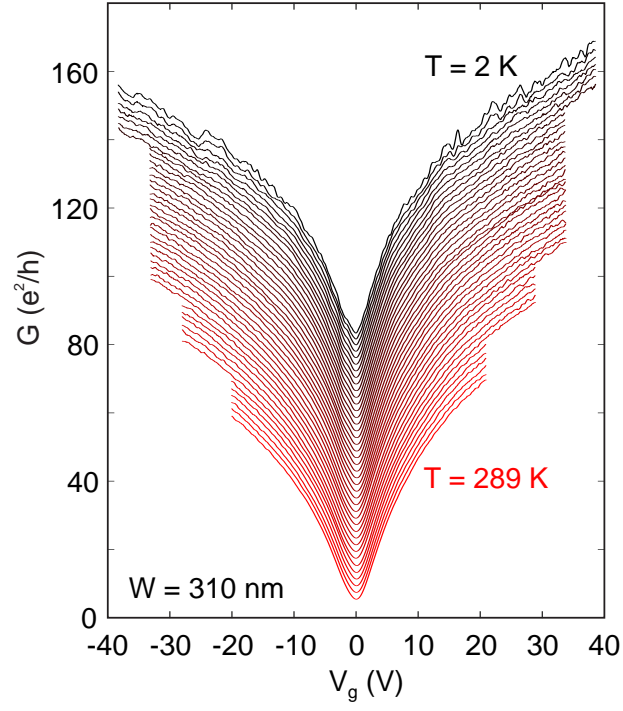
Supplementary Figure S11. Finite bias spectroscopy of the 230 nm-wide graphene constriction. (a) Differential conductance g as a function of back-gate voltage, measured at $V_b = 0$ V, $B = 0$ T and $T = 6$ K. The red solid line shows the ballistic model of conductance, fitted at high carrier densities. (b) Differential conductance g as a function of source-drain voltage V_b . The traces are taken at fixed values of back-gate voltage V_g from -0.5 V (lower trace) to 3.0 V (upper trace) in steps of 30 mV. The dense regions correspond to kinks in conductance. The intermediate kinks at high bias voltage are marked by red arrows. The subband spacing $\Delta E \approx 13.5 \pm 3$ meV is highlighted by a vertical red line. (c) Differential conductance g as a function of source-drain voltage V_b measured at $B = 140$ mT. The intermediate kinks at high bias voltage are marked by red arrows. We extract an equal subband spacing as in panel b, $\Delta E \approx 13.5 \pm 3$ meV (vertical red line).



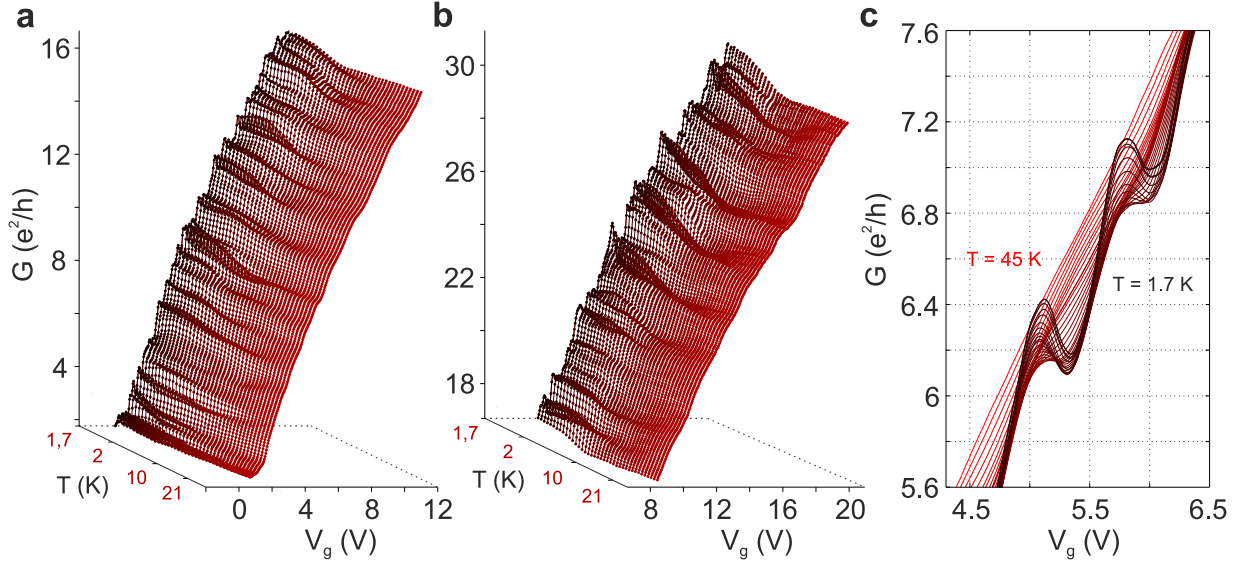
Supplementary Figure S12. Finite DC bias spectroscopy of the 230 nm-wide graphene constriction. (a) DC spectroscopy of the same device as in Figure S11. (b) Conductance G as a function of DC source-drain voltage V_b measured at $B = 140$ mT and $T = 6$ K, for the same device as in panel a.

Supplementary Note 7. Temperature dependence

In this note we show additional data on the temperature dependence of our transport data highlighting both (i) the high quality of our samples and (ii) the energy scale and stability of the observed kink features.



Supplementary Figure S13. Temperature dependence of the back-gate characteristics for the 310 nm-wide graphene constriction. Low-bias back-gate dependent four-terminal conductance G as a function of temperature T . The traces are shifted in the conductance axis for clarity. Temperature is recorded from $T = 2$ K (black trace) up to room-temperature ($T = 289$ K, red trace), in steps of 7 K.



Supplementary Figure S14. Temperature dependence of the conductance kinks for the 230 nm-wide constriction. (a) and (b) Four-terminal conductance G as a function of back gate voltage V_g and temperature T , at low (panel a) and high (panel b) carrier densities. Measurements are recorded at temperatures from $T = 2$ K to $T = 24$ K in steps of 0.7 K. (c) Zoom-in of the temperature evolution of the shape of the kinks.

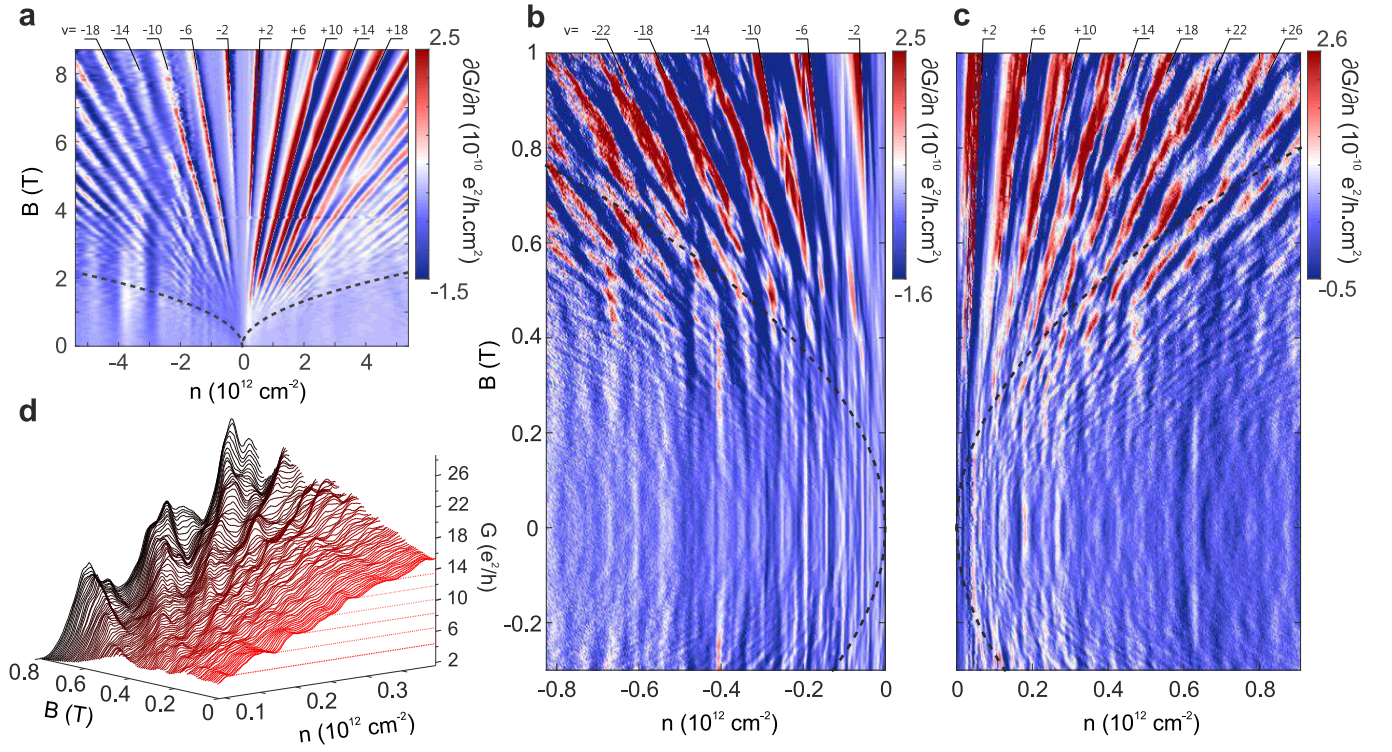
Supplementary Note 8. Evolution of size quantization with magnetic field

We provide an additional data set for the magnetic-field evolution of the size quantization signatures from the 280 nm-wide graphene constriction in Fig. S15. We find the same transition from size-quantization signatures, at low magnetic fields, to the Landau level regime, at high magnetic fields, as in the sample discussed in the main text (see Fig. 5 of main manuscript).

Supplementary Note 9. Theoretical treatment

We use a third nearest neighbor tight-binding approach to simulate the constriction. We pattern the device edge using the experimental geometry determined from SEM, and a correlated random fluctuation to simulate microscopic roughness. We rescale our device by a factor of four compared to the experiment, to arrive at a numerically feasible system size. Such a rescaling by a factor of four ensures that all relevant length scales of the problem (e.g., device geometry, Fermi wavelength, magnetic length and correlation length of the edge roughness) are still much larger than the discretization length of the numerical graphene lattice, allowing to extrapolate simulation data to the experimental result⁷. We use a correlation length of 5 nm and an average disorder amplitude of 13 nm. We determine the Green's function, $\mathcal{G}(r, r')$, of the device using the modular recursive Green's function method^{8,9}. The local density of states, $\rho(r, E)$, is given by $\rho(r, E) \propto \text{Im}[\mathcal{G}(r, r; E)]$. Calculations were performed on the Vienna Scientific Cluster 3. To determine the transport properties of the device, we attach two leads of width D on each side of the experimental contact regions, and calculate the total transmission. To avoid residual effects due to the fixed lead width used in the computation, we average over five different randomly chosen lead widths $D \in [60, 80]$ nm.

To determine the evolution of subbands in a constriction of width W with magnetic field, we calculate the band structure of a perfect zigzag graphene nanoribbon of width W as a function of magnetic field. We include the magnetic field via a Peierls phase factor. The subband positions are extracted from the minima of each band in the bandstructure of the ribbon.



Supplementary Figure S15. Magnetic-field dependence of the size quantization for the 280 nm-wide graphene constriction. (a) Landau level fan of the 280 nm-wide graphene constriction. (b) and (c) High resolution double derivative plots, measured at low magnetic fields $B \leq 1$ T, in the low-carrier density range for the hole- and electron-regimes, respectively. In panels a, b and c the black dashed line denotes the boundary above which the magnetic field quantization of Landau level m dominates over size quantization, i.e. when $2\sqrt{2m}l_B < W$. (d) Evolution of the conductance traces as a function of charge carrier density n and magnetic field B . The B -field step size between traces is 8 mT. The data was measured at $T = 1.7$ K.

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